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HYPERSONIC RESEARCH PROJECT

Memorandum No. 36

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ARMY ORDNANCE CONTRACT NO. DA-04-495-Ord-19

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CALIFORNIA INSTITUTE OF TECHNOLOGY
Pasadena, California

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ABSTRACT

The thirteen-moment approximation developed by H. Grad for solving the Maxwell-Boltzmann equation is applied to the problem of the relative shearing motion between two infinite, parallel flat plates (plane Couette flow). In order to bring out the molecular effects as directly as possible the problem is linearized by requiring that the Mach number is small compared with unity, and that the temperature difference between the two plates is small compared with ambient temperature. According to the linearized Grad equations the shear stress in this case is given by the usual Navier-Stokes relation for all values of the parameter Re/M, in agreement with R. A. Millikan's postulate. Also the linearized boundary conditions for this problem are identical with the Maxwell slip relations utilized by Millikan, so the same expressions for slip velocity and drag coefficient are obtained. An examination of the drag data obtained by Kuhlthau, Chiang, and Bowyer and Talbot in their rotating-cylinder experiments at low densities shows that the variation of $1/C_D M$ with Re/M is predicted reasonably well by this theory over a range of Mach numbers from 0.15 to 1.40, in spite of the fact that the theory is supposed to hold only for low Mach numbers.

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LIST OF SYMBOLS

a	speed of sound
C_D	drag coefficient
C_p, C_v	specific heat at constant pressure and constant volume, respectively
d	distance between the upper and the lower plates
f	$1 - \alpha$
k	coefficient of heat conduction
L	mean free path in the gas
M	Mach number, U/a
p	pressure of gas
p_{ij}	stress (increment over hydrostatic pressure)
Pr	Prandtl number $= \frac{C_p \mu}{k}$
q_i	heat flux
r_1, r_2	radii of inner and outer cylinders, respectively
R	gas constant
Re	Reynolds number $\frac{\rho U d}{\mu}$ or $\frac{\rho\, U(r_2 - r_1)}{\mu}$
T	temperature of gas
U	velocity of upper plate in its own plane
u, v	velocity components parallel and normal to plates, respectively
x, y	coordinates parallel and normal to the plates, respectively
α	fraction of incident gas molecules specularly reflected from the plate
γ	ratio of specific heats $= C_p/C_v$
μ	coefficient of viscosity
ρ	density of gas

Subscripts

0	quantities in the gas near the lower plate
1	quantities on the lower plate
2	quantities on the upper plate

Prime

'	perturbation quantities

I. INTRODUCTION

Because of the difficulties involved in solving the Maxwell-Boltzmann equation, every theoretical analysis of low density gas flows so far proposed is based on certain simplifying approximations. The nature of these approximations is brought out most clearly by applying the proposed schemes to some simple flow problems, for which the Navier-Stokes solutions are already well-known. From this point of view the flow generated by the relative shear motion of two infinite, parallel flat plates (plane Couette flow) is a natural testing ground. In 1923 R. A. Millikan[1] boldly proposed that the Navier-Stokes relation $p_{xy} = \mu(du/dy)$ should hold also in the so-called slip-flow regime, and that the slip velocity at either plate surface is given by Maxwell's simple formula

$$|\Delta u| = \left(\frac{2-f}{f} \right) L \, (du/dy) \quad ,$$

where f is the fraction of incident molecules reflected diffusely, and L is the mean free path in the gas. Because the inertial forces are zero in this case, p_{xy} = const., and Millikan was able to obtain a simple expression for the drag coefficient on either plate that reduces to the free-molecule flow value at extremely low pressures ($Re/M \longrightarrow 0$), and to the usual Navier-Stokes relation at high pressures ($Re/M \longrightarrow \infty$). This scheme can also be applied to the problem of heat conduction between two plates.[2] Millikan's solution is valid for the closely-related problem of the flow generated by two concentric rotating cylinders (for which it was originally intended), provided that the ratio of the distance between the cylinders to the radius of either cylinder is small.

It is remarkable that Millikan's result, which was supposed to hold only in the slip-flow regime, agrees well with all the available experimental data for the drag on a rotating cylinder over a very wide range of gas density, or Re/M. Yet no satisfactory theoretical justification for this behavior has ever been given. Schamberg[3] utilized the Chapman-Enskog-Burnett scheme of expanding the stresses and heat flux in powers of $(\mu/p)(U/d) = M^2/Re$, and found that the second-order or Burnett terms vanished identically in steady, plane Couette flow. In other words, the Navier-Stokes and Fourier relations hold up to (but not including) terms of the third order. However, the slip velocity at either plate surface introduces terms of order $L/d \sim M/Re$, $(M/Re)^2$, and also a term of order M^4/Re^2 from the molecular velocity distribution function. Schamberg's expression for the drag coefficient is identical with the series expansion of Millikan's result in powers of M/Re up to terms of order $(M/Re)^2$. This fact led Chiang[4] to conclude that the validity of Millikan's expression in the slip-flow regime* should be expected, at least for $M < 1$. But serious doubts have been raised recently regarding the validity of the whole Chapman-Enskog-Burnett expansion scheme (See Reference 5, for example), and the question remains unsettled. At any rate Schamberg's results are certainly not applicable when $M/Re = 0(1)$, or larger.

A more satisfactory approach to the problem of low-density

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* M/Re small compared with unity, but not negligible.

gas flow was proposed by H. Grad.[6] In Grad's scheme the departure of the molecular velocity distribution function from local Maxwellian is expressed as a series expansion in Hermite polynomials involving the molecular velocities, and the coefficients of this series are identified with the stresses, heat fluxes, and higher moments. In the thirteen-moment approximation only the stresses and heat fluxes appear, and these quantities are regarded as new variables not explicitly related to the gradients of mean velocity and temperature. Differential equations for these quantities are obtained by substituting Grad's distribution function into the Maxwell-Boltzmann equation, and then requiring that this equation be approximately satisfied. Chiang[4] attempted to apply Grad's scheme to both plane and cylindrical Couette flow, but Grad's system of equations is highly non-linear and no general solutions could be found. Instead Chiang obtained solutions for the slip-flow regime by expanding in powers of M^2/Re. He showed that the second order terms in the expansion for the shear stress and heat flux are identically zero, as in the Burnett expansion*, but that third and higher order terms now appear involving $(du/dy)^2$, $(du/dy)^2\,(d^2T/dy^2)$, etc. Again the results for drag coefficient and slip velocity are identical with Millikan's expressions up to terms of order $(M/Re)^2$. In other words nothing essentially new is obtained, and this scheme evidently fails to take advantage of the fact that Grad's method does not imply any explicit local relation between stresses and velocity gradients, and should therefore be applicable over the

* In a paper to appear shortly, H. T. Yang and T. Y. Li have shown that Grad's method always includes the Burnett procedure as a special case when M/Re is small compared with unity.

whole range of values of the parameter Re/M.

At the opposite end of the density scale, Wang Chang and Uhlenbeck[7] attempted to solve the problem of plane Couette flow by means of a series solution of the linearized Maxwell-Boltzmann equation. They obtained an expression for the shear stress of the form

$$p_{xy} = p_{xy}^{(0)} \left[1 + \beta_1 \ d/L + \ldots \right] \quad ,$$

where $p_{xy}^{(0)}$ is the free-molecule flow value for diffuse reflection. In a later paper[8] these authors discovered serious divergence difficulties connected with such an expansion procedure for this particular case. Physically, molecules that reflect from either plate at nearly glancing angles traverse very long distances before reaching the other plate, so that for finite L some collisions in the gas must certainly occur. The authors state that these difficulties disappear in the case of the Couette flow between two concentric rotating cylinders.* In other words two distinct limiting processes are involved here. In one limiting procedure the ratio of the spacing between cylinders, $r_2 - r_1$, to the radius of the inner cylinder r_1, is first held <u>fixed</u>, while $\rho \longrightarrow 0$, $L \longrightarrow \infty$, and $Re/M \longrightarrow 0$. <u>Then</u> $(r_2 - r_1)/r_1$ is allowed to approach zero. This process gives no difficulties. But Wang Chang and Uhlenbeck adopted the alternative procedure of first allowing $(r_2 - r_1)/r_1 \longrightarrow 0$ (flat plates) with L finite, and <u>then</u> permitting $L \longrightarrow \infty$. Evidently these two limiting procedures

* Unfortunately no written version of their results for cylindrical Couette flow has ever appeared.

applied to the Maxwell-Boltzmann equation are not interchangeable.*

In Grad's method the basic equations already contain the results of an averaging process applied to all molecules, and no divergence difficulties are expected to appear in the case of two infinite, parallel flat plates. The point of view adopted by the present authors is that Grad's method offers a consistent theoretical framework for the whole range of gas densities, at least for shear flows. Because of the difficulty of solving the full non-linear Grad equations the problem of plane Couette flow is now linearized by requiring that (1), the velocity of the upper plate is small compared with the ambient sound speed; and (2), the difference in temperature between the two plates is small compared with the ambient temperature. In other words the problem is purposely simplified so that the molecular effects are brought out as directly as possible. This linearization procedure has already been successfully utilized by the present authors for the Rayleigh problem.[9] We are also encouraged in this attempt by the experimental results obtained by Kuhlthau[10] for the drag coefficient on a rotating cylinder. He finds that the effect of Mach number is negligible in the range $0.02 \leq Re/M \leq 11.4$ up to the highest rotor Mach number of about 1.40. Chiang[4] also found that the Mach number effect is very small, up to $M = 0.55$. Thus the solutions of the

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* An approximate method of solving the Boltzman equation for plane Couette motion has been developed for the intermediate region between free molecule and Navier-Stokes flow by P. Gross, E. A. Jackson, and S. Ziering of Syracuse University, but only a brief abstract is available to the authors. ["Kinetic Theory of Couette Flow", Bulletin of the American Physical Society, Series II, Volume 1, No. 4, p. 227, April 26, 1956.]

linearized problem should furnish a useful first approximation for subsonic and low supersonic flow.*

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* Grad's equations have been applied to the problem of plane Couette flow by Grad[11], and to cylindrical Couette flow by Grad's student M. Rose[12], but these results were never published. In a recent discussion Dr. Grad informed one of the present authors that explicit solutions of the plane Couette flow problem were not obtained, and that there was also some difficulty with the expansion in powers of $1/r_1$ for the rotating cylinder problem. It is hoped that the work of Grad and Rose will appear shortly.

II. GENERAL EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

Consider the problem of the gas flow between two infinite, parallel plates at a distance d apart, with the lower plate stationary at a temperature T_1, and the upper plate moving with a velocity U in its own plane at a temperature $T_2 < T_1$. It is assumed that sufficient time has elapsed since the start of the motion of the upper plate so that the gas between the plates has reached a steady state; therefore, the only independent variable involved is the ordinate y normal to the plates. The coordinate system is fixed on the stationary lower plate as shown in Figure 1.

For this problem the equations of motion developed by H. Grad[6] take the following form: [for a monatomic gas $\gamma = C_p/C_v = 5/3$ and $Pr = (C_p \mu/k) = 2/3$].

Continuity

$$v \frac{d\rho}{dy} + \rho \frac{dv}{dy} = 0 \tag{2.1}$$

Momentum

$$v \frac{du}{dy} + \frac{1}{\rho} \frac{dp_{xy}}{dy} = 0 \tag{2.2}$$

$$v \frac{dv}{dy} + \frac{1}{\rho} \frac{dp}{dy} + \frac{1}{\rho} \frac{dp_{yy}}{dy} = 0 \tag{2.3}$$

Energy

$$v \frac{dp}{dy} + \frac{2}{3} p_{xy} \frac{du}{dy} + \left(\frac{5}{3} p + \frac{2}{3} p_{yy}\right) \frac{dv}{dy} + \frac{2}{3} \frac{dq_y}{dy} = 0 \tag{2.4}$$

Stress

$$v \frac{dp_{xx}}{dy} + \frac{4}{3} p_{xy} \frac{du}{dy} + \left(p_{xx} - \frac{2}{3} p_{yy} - \frac{2}{3} p\right) \frac{dv}{dy} - \frac{4}{15} \frac{dq_y}{dy} + \frac{p}{\mu} p_{xx} = 0 \tag{2.5}$$

$$v \frac{dp_{xy}}{dy} + (p + p_{yy}) \frac{du}{dy} + 2 p_{xy} \frac{dv}{dy} + \frac{2}{5} \frac{dq_x}{dy} + \frac{p}{\mu} p_{xy} = 0 \tag{2.6}$$

$$v \frac{dp_{yy}}{dy} + \left(\frac{4}{3} p + \frac{7}{3} p_{yy} \right) \frac{dv}{dy} - \frac{2}{3} p_{xy} \frac{du}{dy} + \frac{8}{15} \frac{dq_y}{dy} + \frac{p}{\mu} p_{yy} = 0 \tag{2.7}$$

Heat Flux

$$v \frac{dq_x}{dy} + \frac{7}{5} q_y \frac{du}{dy} + \frac{7}{5} q_x \frac{dv}{dy} + \frac{5}{2} \frac{p_{xy}}{\rho} \frac{dp}{dy} - \frac{7}{2} \frac{p\, p_{xy}}{\rho^2} \frac{d\rho}{dy} + \left(\frac{p}{\rho} - \frac{p_{xx}}{\rho} \right) \frac{dp_{xy}}{dy} - \frac{p_{xy}}{\rho} \frac{dp_{yy}}{dy} + \frac{2}{3} \frac{p}{\mu} q_x = 0 \tag{2.8}$$

$$v \frac{dq_y}{dy} + \frac{16}{5} q_y \frac{dv}{dy} + \frac{2}{5} q_x \frac{du}{dy} + \frac{5}{2} \left(\frac{p}{\rho} + \frac{p_{yy}}{\rho} \right) \frac{dp}{dy} - \left(\frac{5}{2} \frac{p^2}{\rho^2} + \frac{7}{2} \frac{p\, p_{yy}}{\rho^2} \right) \frac{d\rho}{dy} + \left(\frac{p}{\rho} - \frac{p_{yy}}{\rho} \right) \frac{dp_{yy}}{dy}$$
$$- \frac{p_{xy}}{\rho} \frac{dp_{xy}}{dy} + \frac{2}{3} \frac{p}{\mu} q_y = 0 \tag{2.9}$$

Equation of state

$$p = \rho R T \tag{2.10}$$

The boundary conditions[9] for this problem are as follows:

At the lower plate (y = 0)

$$v(0) = 0 \tag{2.11}$$

$$\left[\frac{2\pi}{RT(0)} \right]^{1/2} \frac{q_y(0)}{p(0)} + \frac{4(1-\alpha)}{1+\alpha} \left[1 - \frac{T_1}{T(0)} + \frac{p_{yy}(0)}{2p(0)} \left(\frac{3}{2} - \frac{T_1}{T(0)} \right) - \left(1 + \frac{p_{yy}(0)}{2p(0)} \right) \frac{\left(\frac{u(0)}{2} \right)^2}{RT(0)} \right] = 0 \tag{2.12}$$

$$\frac{p_{xy}(0)}{p(0)} + \frac{2(1-\alpha)}{1+\alpha} \frac{u(0)}{[2\pi RT(0)]^{\frac{1}{2}}} \left[1 + \frac{p_{yy}(0)}{2p(0)} \right] + \frac{2(1-\alpha)}{5(1+\alpha)} \frac{q_x(0)}{p(0)\,[2\pi RT(0)]^{1/2}} = 0 \tag{2.13}$$

where α is the fraction of molecules specularly reflected. *

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* In the final results we take $\alpha = 0$ (diffuse reflection)

At the upper plate (y = d)

$$v(d) = 0 \tag{2.14}$$

$$\left[\frac{2\pi}{RT(d)}\right]^{\frac{1}{2}} \frac{q_y(d)}{\rho(d)} + \frac{4(1-\alpha)}{1+\alpha}\left[1 - \frac{T_2}{T(d)} + \frac{p_{yy}(d)}{2\rho(d)}\left(\frac{3}{2} - \frac{T_2}{T(d)}\right) - \left(1 + \frac{p_{yy}(d)}{2\rho(d)}\right)\frac{\left(\frac{u(d)-U}{2}\right)^2}{RT(d)}\right] = 0 \tag{2.15}$$

$$\frac{p_{xy}(d)}{\rho(d)} + \frac{2(1-\alpha)}{1+\alpha}\frac{u(d)-U}{[2\pi RT(d)]^{\frac{1}{2}}}\left[1 + \frac{p_{yy}(d)}{2\rho(d)}\right] + \frac{2(1-\alpha)}{5(1+\alpha)}\frac{q_x(d)}{\rho(d)[2\pi RT(d)]^{1/2}} = 0 \tag{2.16}$$

From Eq. (2.1) and the boundary conditions (2.11) and (2.14), we see that

$$\rho v = 0.$$

Since $\rho \neq 0$

$$v \equiv 0 \tag{2.17}$$

i.e., the gas flow is parallel to the plates.

With v = 0 in Eqs. (2.2), (2.3), and (2.4), we have

$$p_{xy} = \text{const} \tag{2.18}$$

$$p + p_{yy} = \text{const} \tag{2.19}$$

$$p_{xy}\, u + q_y = \text{const} \tag{2.20}$$

By substituting Eq. (2.4) into Eqs. (2.5) and (2.7), and then comparing, we find

$$p_{xx} = -(4/3)\, p_{yy} \tag{2.21}$$

So far these results are valid for general plane Couette flow without any additional assumptions. But the system of equations is highly non-linear, and the treatment is now restricted to the simplified case of small plate velocity and small temperature difference between the plates.

III. LOW-SPEED PLANE COUETTE FLOW

When the velocity of the upper plate is small and the temperature difference between the two plates is also small; or more precisely, when

$$M = U/a_o << 1$$

and

$$(T_1 - T_2)/T_o << 1$$

Then

$$\begin{aligned} \rho &= \rho_o + \rho' \\ p &= p_o + p' \\ T &= T_o + T' \end{aligned} \tag{3.1}$$

and $Q = Q'$, where Q denotes any velocity component, stress, or heat flux quantity. [Here the primed quantities denote small perturbations.] By substituting Eq. (3.1) into Eqs. (2.1) to (2.16) and neglecting squares and products of all small perturbations the equations of motion as well as the boundary conditions are linearized.* Furthermore, the tangential quantities u, p_{xy}, and q_x, and the normal quantities v, ρ, p (or T), p_{xx}, p_{yy}, and q_y are separated. This uncoupling of momentum and energy is typical of the linearization procedure and is also found in Reference 9.

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* This linearization procedure can be regarded as the first step in a perturbation scheme in the parameters

$$M^2 \text{ and } \frac{T_1 - T_2}{T_o}$$

A. Shear Stress, Tangential Velocity, and Tangential Energy Flux

The tangential quantities are governed by the following relations obtained from Eqs.(2.18), (2.6), and (2.8).

$$p_{xy} = \text{const} \tag{3.2}$$

$$p_{xy} = -\mu_o \, (du/dy) \tag{3.3}$$

$$q_x = 0 \quad \text{(no tangential heat flux)} \tag{3.4}$$

According to the linearized Grad equations for plane Couette flow, the shear stress is given by the Navier-Stokes relation, and this result, which is identical with Millikan's postulate[1], is valid for all values of the parameter Re/M.

The boundary conditions are obtained from Eqs. (2.13) and (2.16) as follows:

$$\frac{p_{xy}(0)}{p_o} + \frac{2(1-\alpha)}{1+\alpha} \frac{u(0)}{[2\pi RT(0)]^{1/2}} = 0 \tag{3.5}$$

$$\frac{p_{xy}(d)}{p_o} + \frac{2(1-\alpha)}{1+\alpha} \frac{u(d)-U}{[2\pi RT(d)]^{1/2}} = 0 \tag{3.6}$$

Clearly the boundary conditions (3.5) and (3.6) in this case are identical with the Maxwell slip relations utilized by Millikan, and so the same expressions for the slip velocity and drag coefficient on either plate are obtained. By referring to Figure 2, we see that

$$\frac{du}{dy} = \frac{u(d)-u(0)}{d} = -\frac{p_{xy}}{\mu_o} = \text{const} \tag{3.7}$$

or

$$u(y) = \frac{u(d)-u(0)}{d} \, y + u(0) \tag{3.8}$$

From the boundary conditions (3.5) and (3.6), we have finally

$$\frac{u(y)}{U} = \frac{1}{1+\frac{1+\alpha}{1-\alpha}\frac{\mu_0\sqrt{2\pi R T_0}}{\rho_0 d}}\frac{y}{d} + \frac{1}{2}\frac{1}{1+\frac{1-\alpha}{1+\alpha}\frac{\rho_0 d}{\mu_0\sqrt{2\pi R T_0}}} \tag{3.9}$$

$$= \frac{1}{1+\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\frac{M}{Re}}\frac{y}{d} + \frac{1}{2}\frac{1}{1+\frac{1-\alpha}{1+\alpha}\sqrt{\frac{3}{10\pi}}\frac{Re}{M}}$$

and

$$p_{xy} = -\mu_0 \frac{1}{1+\frac{1+\alpha}{1-\alpha}\frac{\mu_0\sqrt{2\pi R T_0}}{\rho_0 d}}\cdot\frac{U}{d} \tag{3.10}$$

$$= -\mu_0 \frac{1}{1+\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\frac{M}{Re}}\cdot\frac{U}{d}$$

where

$$M = \text{Mach number} = \frac{U}{\sqrt{\frac{5}{3}RT_0}}$$

$$Re = \text{Reynolds number} = \frac{\rho_0 U d}{\mu_0}$$

From Eqs. (3.9) and (3.10) we obtain the most interesting quantities of velocity slip and drag coefficient. Thus

$$\frac{u(0)}{U} = 1 - \frac{u(d)}{U} = \frac{1}{2}\frac{1}{1+\frac{1-\alpha}{1+\alpha}\sqrt{\frac{3}{10\pi}}\frac{Re}{M}} \tag{3.11}$$

$$C_D M = M\frac{-p_{xy}}{\frac{1}{2}\rho_0 U^2} = \frac{2}{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}+\frac{Re}{M}} \tag{3.12}$$

or,

$$\frac{1}{C_D M} = \frac{1}{2}\left[\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}+\frac{Re}{M}\right] \tag{3.13}$$

Note that the term $\frac{1}{2}$ Re/M is independent of α and is also independent of the nature of the gas.

In the limiting case $Re/M \longrightarrow \infty$,

$$\frac{u(0)}{U} = 1 - \frac{u(d)}{U} = 0 \qquad (3.14)$$

$$C_D = 2/Re \qquad (3.15)$$

i.e., there is no velocity slip and the skin friction is given by the classical Navier-Stokes formula. In the opposite limiting case of free-molecule flow $Re/M \longrightarrow 0$

$$\frac{u(0)}{U} = 1 - \frac{u(d)}{U} = \frac{1}{2} \qquad (3.16)$$

$$C_D = \frac{1-\alpha}{1+\alpha} \sqrt{\frac{6}{5\pi}} \frac{1}{M} \qquad (3.17)$$

i.e., there is a 50 per cent velocity slip (regardless of the value of α, so long as it is not equal to 1*), and the expression for C_D is identical with the free molecule flow result. The variation of $u(0)/U$ and $1/(C_D M)$ with Re/M for a diffusively reflecting surface ($\alpha = 0$) is plotted in Figures 3 and 4, respectively.

B. Thermodynamic Quantities, Normal Stresses, and Normal Heat Flux

After linearization the energy equation [Eq. (2.4)] states that $dq_y/dy = 0$, or $q_y = \text{const.}$, so that $p_{xx} = p_{yy} = 0$ according to Eq. (2.5) and (2.7), and $p = p_o$ by Eq. (2.3) or (2.19). In addition $v \equiv 0$, as shown in Section II. By Eq. (2.9) and (2.10),

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* The case of $\alpha = 1$ (i.e., specularly reflecting surface) corresponds to the special situation in which the motion of the upper plate can never be transmitted to the gas.

$$q_y = -\frac{15}{4}\,\mu_o R\,\frac{dT'}{dy} = -\frac{3}{2}\,c_p\mu_o\,\frac{dT'}{dy} = -k_o\,\frac{dT'}{dy} \tag{3.18}$$

$$\rho' = -(\rho_o/T_o)\,T' \tag{3.19}$$

The boundary conditions obtained from Eqs. (2.12) and (2.15) are as follows:

$$\left(\frac{2\pi}{RT(0)}\right)^{\frac{1}{2}}\frac{q_y(0)}{p_o} + \frac{4(1-\alpha)}{1+\alpha}\left[1-\frac{T_1}{T(0)}\right] = 0 \tag{3.20}$$

$$\left(\frac{2\pi}{RT(d)}\right)^{1/2}\frac{q_y(d)}{p_o} + \frac{4(1-\alpha)}{1+\alpha}\left[1-\frac{T_2}{T(d)}\right] = 0 \tag{3.21}$$

Here again the heat flux q_y is given by the Fourier-Newton relation for all values of the parameter Re/M. The heat flux is constant because the pressure work and dissipation terms are neglected.

From Figure 5, we have

$$\frac{dT'}{dy} = \frac{T(d)-T(0)}{d} = -\frac{q_y}{k_o} = \text{const.}$$

or

$$T'(y) = \frac{T(d)-T(0)}{d}\,y \quad\text{and}\quad T(y) = T(0) + \frac{T(d)-T(0)}{d}\,y \tag{3.22}$$

By utilizing the boundary conditions (3.20) and (3.21), we have finally

$$T(y) = \frac{1+\frac{1+\alpha}{4(1-\alpha)}\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{k_0(T_1+T_2)}{p_0 d}}{1+\frac{1+\alpha}{2(1-\alpha)}\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{k_0 T_1}{p_0 d}}\,T_1 - \frac{T_1 - T_2}{1+\frac{1}{2}\frac{(1+\alpha)}{(1-\alpha)}\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{k_0 T_1}{p_0 d}}\,\frac{y}{d}$$

$$= \frac{1+\frac{5(1+\alpha)}{8(1-\alpha)}\sqrt{\frac{10\pi}{3}}\left(1+\frac{T_2}{T_1}\right)\frac{M}{P_r Re}}{1+\frac{5(1+\alpha)}{4(1-\alpha)}\sqrt{\frac{10\pi}{3}}\frac{M}{P_r Re}}\,T_1 - \frac{T_1 - T_2}{1+\frac{5(1+\alpha)}{4(1-\alpha)}\sqrt{\frac{10\pi}{3}}\frac{M}{P_r Re}}\,\frac{y}{d} \qquad (3.23)$$

$$q_y = k_0\frac{T_1 - T_2}{1+\frac{1+\alpha}{2(1-\alpha)}\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{kT_1}{p_0 d}}\,\frac{1}{d}$$

$$= k_0\frac{T_1 - T_2}{d}\,\frac{1}{1+\frac{5}{4}\frac{(1+\alpha)}{(1-\alpha)}\sqrt{\frac{10\pi}{3}}\frac{M}{P_r Re}} \qquad (3.24)$$

$$\rho(y) = \rho_0\left[1+\frac{1-\frac{T_2}{T_1}}{1+\frac{1+\alpha}{2(1-\alpha)}\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{k_0 T_1}{p_0 d}}\,\frac{y}{d}\right]$$

$$= \rho_0\left[1+\frac{1-\frac{T_2}{T_1}}{1+\frac{5(1+\alpha)}{4(1-\alpha)}\sqrt{\frac{10\pi}{3}}\frac{M}{P_r Re}}\,\frac{y}{d}\right] \qquad (3.25)$$

From Eqs. (3.23) and (3.24), we obtain the temperature jump and Stanton number at either plate surface:

$$\frac{T_1 - T(0)}{T_1 - T_2} = \frac{T(d) - T_2}{T_1 - T_2} = \frac{1}{2}\,\frac{1}{1+\frac{4}{5}\frac{1-\alpha}{1+\alpha}\sqrt{\frac{3}{10\pi}}\frac{P_r Re}{M}} \qquad (3.26)$$

$$C_H M = \frac{q_y M}{\rho C_p U (T_1 - T_2)} = \frac{1}{P_r\frac{Re}{M} + \frac{5}{4}\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}} \qquad (3.27)$$

or

$$\frac{1}{C_H M} = \frac{5}{4}\,\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}} + P_r\frac{Re}{M} \qquad (3.28)$$

In the limiting case $Re/M \longrightarrow \infty$

$$T_1 - T(0) = T(d) - T_2 = 0 \tag{3.29}$$

$$q_y = k \frac{T_1 - T_2}{d} \quad , \quad \text{or} \quad C_H = \frac{1}{Pr\, Re} \quad , \tag{3.30}$$

i.e., there is no temperature jump and the Stanton number takes on the well-known value given by the Navier-Stokes solution. For the limiting case of free molecule flow $Re/M \longrightarrow 0$,

$$T_1 - T(0) = T(d) - T_2 = \frac{T_1 - T_2}{2} \tag{3.31}$$

$$C_H = \frac{4}{5} \frac{1-\alpha}{1+\alpha} \sqrt{\frac{3}{10\pi}} \frac{1}{M} \tag{3.32}$$

which is again in agreement with the free molecule flow results.

The variation of

$$\frac{T_1 - T(0)}{T_1 - T_2} \quad \text{and} \quad \frac{1}{C_H M} \quad \text{vs} \quad Re/M$$

for diffusively reflecting surfaces ($\alpha = 0$) is plotted in Figures 3 and 6, respectively. The velocity and temperature distributions for free-molecular, intermediate, and Navier-Stokes plane Couette flow at low Mach number are shown in Figure 7.

IV. COMPARISON BETWEEN THEORY AND ROTATING CYLINDER EXPERIMENTS

A. Relation Between Plane Couette Flow and Cylindrical Couette Flow with Small Gap Ratio

In all of the rotating cylinder experiments at low densities the ratio of the spacing between cylinders, $r_2 - r_1$, to the inner cylinder radius, r_1, is small, and usually no correction is applied to the theoretical results for plane Couette flow when theory and experiment are compared. However, Bowyer and Talbot[13] show that the correction factor that must be applied to the theoretical drag coefficient for plane flow to obtain the value of C_D on the outer fixed cylinder is very nearly $(r_1/r_2)^2$ in free-molecule flow.* In the opposite limiting case of high density, or $Re/M \longrightarrow \infty$, the correction factor is $2r_1/(r_1 + r_2)$, according to Lamb[14]. Thus, a reasonably accurate representation of the drag coefficient on the outer fixed cylinder is given by the following expression derived from Eq. (3.13), with $\alpha = 0$.

$$\frac{1}{C_D M} = \frac{1}{2} \frac{r_2^2}{r_1^2} \sqrt{\frac{10\pi}{3}} + \frac{1}{2} \left(1 + \frac{1}{2} \frac{r_2 - r_1}{r_1}\right) \cdot \frac{Re}{M} \tag{4.1}$$

Strictly speaking the theoretical results obtained by solving the Grad equations are valid only for monatomic gases. But with

---

* Since the torque is constant, $C_D r^2$ = const., and the Bowyer and Talbot result means that the drag coefficient on the inner rotating cylinder is identical with the free-molecule flow value for plane Couette motion. For diffusely reflecting surfaces the inner cylinder "sees" only molecules coming from the outer fixed cylinder with zero mean velocity, and the curvature effect on the inner cylinder is therefore nil. This result is also stated by Kuhlthau[10].

vanishingly small temperature differences in the gas, and also between the gas and the solid surfaces, there is some justification for extending these results to polyatomic gases. In that case the factor $\sqrt{10\pi/3}$ in Eqs. (3.13) and (4.1) is replaced by $\sqrt{2\pi\gamma}$, according to the boundary conditions (3.5) and (3.6).

In the experiments of Bowyer and Talbot[13], $r_1 = 4.25$ in. and $r_2 = 4.75$ in., so that for monatomic gases Eq. (4.1) becomes

$$1/C_D M = 2.02 + 0.53 \, (Re/M) \tag{4.2}$$

This predicted variation of drag coefficient with Re/M is shown in Figure 4. For comparison we have also plotted the drag coefficient obtained by applying the factor r_2^2/r_1^2 to the entire expression in Eq. (3.13), namely,

$$1/C_D M = 2.02 + 0.63 \, (Re/M) \tag{4.3}$$

The experimental results at very low densities ($Re/M \leqq 1$) will also be compared with the Wang Chang-Uhlenbeck[7] expression for $1/C_D M$, corrected by the factor r_2^2/r_1^2, or

$$1/C_D M = 2.02 \left[1 - 0.62 \, \beta \, (Re/M) \right] \tag{4.4}$$

where $\beta_1 = -0.734$ for Maxwellian molecules, and $\beta_1 = -0.870$ for elastic sphere molecules. This theoretical result is also plotted in Figure 4.

In Kuhlthau's experiments[10] $r_1 = 2.0$ in. and $r_2 = 2.47$ in., and the corresponding linearized Grad expression for monatomic gases is (Figure 8b)

$$1/C_D M = 2.47 + 0.56\ (Re/M) \tag{4.5}$$

For the Bowyer and Talbot[13] and Chiang[4] experiments in air, Eq. (4.1) becomes

$$1/C_D M = 1.85 + 0.53\ (Re/M) \tag{4.6}$$

while the Wang Chang-Uhlenbeck expression is

$$1/C_D M = 1.85\ \left[1 - 0.68\ \beta_1\ (Re/M)\right] \tag{4.7}$$

For Kuhlthau's[10] experiments in air the expression for C_D is

$$1/C_D M = 2.26 + 0.56\ (Re/M) \tag{4.8}$$

These three representations of the drag coefficient are shown in Figure 10. For clarity the Wang Chang-Uhlenbeck expression corresponding to Kuhlthau's case is not plotted.

B. Low-Density Experimental Values of the Drag on the Stationary Outer Cylinder in Couette Flow

Measured values of torque obtained by Millikan[1] and others in the rotating-cylinder experiments with a number of gases agreed very closely with Millikan's slip-flow theory, but the lowest value of the parameter Re/M reached in these tests was about 16, and the rotor Mach number was extremely low. It is only recently that experiments have been made in the near-free-molecule regime ($Re/M < 1$), and at high subsonic and supersonic rotor speeds. We will discuss first the experiments on monatomic gases carried out by Bowyer and Talbot[13] and by Kuhlthau[10], and then analyze the experiments in air by these same workers, and also by Chiang[4]

1. Monatomic Gases

a. Bowyer and Talbot[13]

The range of experimental parameters is summarized in the following table:

Gas	Rotor Mach Number	Range of Values of Re/M
Helium	0.047 0.092 0.147 0.223	$0.02 \leqq (Re/M) \leqq 1.0$
Argon	0.149 0.211 0.435	$0.02 \leqq (Re/M) \leqq 1.8$
Krypton	0.214 0.440 0.880 1.325 1.706	$0.05 \leqq (Re/M) \leqq 1.7$

In general the argon experiments exhibited the least scatter and gave the closest agreement with the theoretical prediction. In Figure 4 the experimental values of drag coefficient on the outer fixed cylinder in argon are replotted from Figure 5b and 5c of Reference 13 for rotor Mach numbers of 0.211 and 0.435. The scatter in the data at very low values of Re/M (<0.2) amounts to about $\pm$ 8 per cent, and is indicated by the cross-hatched area. Evidently the simple expression Eq. (4.2) provides an excellent representation of the data at these two Mach numbers. On the other hand the results obtained at a rotor Mach number of 0.149 (not shown) exhibited much larger scatter, and the values of C_D were somewhat lower than either the theory or the experimental values obtained at the two higher Mach numbers.

The experimental results obtained in helium are somewhat more difficult to interpret. In Figure 8a we reproduce the helium data for rotor Mach numbers of 0.047 and 0.092. At least the slope of the average curve of $1/C_DM$ vs. Re/M through the data is in reasonable agreement with the linearized Grad theory, but the drag coefficients are 10 - 15 per cent below both the predicted values and the argon experimental values. These lower values might be attributed to an accommodation coefficient less than unity for helium. (See IV. B. 2.)

In the case of krypton, the drag coefficients are 10 - 15 per cent higher than the free-molecule values at the lowest values of Re/M (Figure 9). However, the data at larger values of Re/M agree reasonably well with the Grad theory prediction*.

To summarize: The Bowyer and Talbot data on monatomic gases and the prediction of the linearized Grad theory are in close agreement in the range $0.02 \leqq Re/M \leqq 2$, for Mach numbers in the range $0.05 \leqq M \leqq 0.44$. In particular the slope of the curve of $1/C_DM$ vs. Re/M is remarkably close to the theoretical value, which is also the value of the slope for $Re/M \longrightarrow \infty$. At the lowest values of Re/M the drag coefficients for helium appear to be about 10 - 15 per cent lower than the free-molecule values, while the krypton values are 10 - 15 per cent higher, as compared with an overall probable error in C_DM of $\pm$ 5 - 8 per cent.

---

* Unfortunately, there is a "break" in the data for $M = 0.214$ at a value of Re/M of about 0.8. Bowyer and Talbot attribute this "break" to the change-over from ion-gage to low-range McLeod gage pressure measurement.

b. Kuhlthau[10]

In Figures 9 and 10 of Reference 10 Kuhlthau presents values of the torque obtained in helium at Mach numbers of 0.128 and 0.512. The mean free path was recalculated utilizing the more modern value of the collision cross-section for helium given by Hirschfelder, namely $\sigma_{He} = 2.576$ Å, and the parameter Re/M is then obtained from the expression $Re/M = 0.203\ p_\mu$, where p_μ is the pressure in microns. Values of $C_D M$ were obtained from the torque data by utilizing the constants of the apparatus and the rotational speed. The results are plotted as $1/C_D M$ vs. Re/M in Figure 8b. Here the average deviation of the drag values from the theoretical prediction is about $\pm$ 5 per cent.

2. Air

The range of experimental parameters reported by each investigator is as follows:

Source	Mach Number Range	Range of Values of Re/M
Kuhlthau[10]	$0.4 \leq M \leq 1.4$	$0.02 \leq Re/M \leq 11.4$
Chiang[4]	$0.15 \leq M \leq 0.55$	$0.38 \leq Re/M \leq 84$
Bowyer and Talbot[13]	$M = 0.147, 0.209, 0.430, 0.852$	$0.04 \leq Re/M \leq 1.0$

In Figure 10 the experimental values of the drag coefficient on the stationary outer cylinder are shown for the lowest pressures so far attained in air. Kuhlthau's values agree remarkably well with the free-molecule flow result at very low values of Re/M. At higher values of Re/M Kuhlthau's data appears to be about 5 - 10 per cent lower than the prediction of the linearized Grad theory, while the Bowyer and Talbot data is some 10 - 15 per cent lower than this theoretical result. Chiang's single data point in this range is quite close to the theory.*

Some of the scatter in the data (and also the fact that the drag values appear to be somewhat low) may perhaps be explained by Merlic's recent discovery[15] that the effective molecular reflection coefficient depends upon the previous pressure history of the apparatus. He found that when the surface was held at a very low pressure of 10^{-2} microns for several days before an experiment was performed, then $f \cong 0.9$ (or $\alpha \cong 0.1$) at a test pressure of 40 microns. On the other hand when the surface was held at 250 microns before the experiment Merlic found that $f \cong 0.6$ at the same test pressure of 40 microns.

---

* Kuhlthau's values of the drag coefficient on the rotor are given in terms of the quantity $C_D \widetilde{Re}$, where $\widetilde{Re}$ is an "effective" Reynolds number chosen so that $C_D \widetilde{Re} \longrightarrow 2$ when $Re/M \longrightarrow \infty$. The true Reynolds number $Re = \frac{\rho U (r_2 - r_1)}{\mu}$ is connected with $\widetilde{Re}$ by the relation

$$Re = \frac{2 r_2^2}{r_1 (r_1 + r_2)} \widetilde{Re}$$

Also the torque is constant, so that $C_{D\ \mathrm{stator}} = (r_1^2/r_2^2)\, C_{D\ \mathrm{rotor}}$. Therefore, $(C_D Re)_{\mathrm{stator}} = \left[2r_1/(r_1 + r_2) \right] (C_D \widetilde{Re})_{\mathrm{rotor}}$, where the values of $(C_D \widetilde{Re})_{\mathrm{rotor}}$ are taken from Figures 11 and 14 of Reference 10. Also the values of $\widetilde{Re}/M$ given by Kuhlthau are corrected to the actual values of Re/M. (Unfortunately Kuhlthau uses the symbol Re for his $\widetilde{Re}$!) This procedure was checked by an independent computation of the Knudsen number $Kn = L/(R_2 - r_1)$; for air $Re/M = 1.48/Kn$.

Merlic suggests that an adsorbed film is formed at the surface and is not removed unless the apparatus is held at very low pressures for some time. This film reflects a certain portion of the molecules specularly. Of course this explanation does not account for the fact that Bowyer and Talbot measured drag coefficients higher than the free-molecule flow values in krypton at very low Re/M.

In Figure 11 the experimental data are plotted over a much wider range, i. e., $0.10 \leqq Re/M \leqq 100$. The agreement between the linearized Grad theory and experiment is good over most of this range. The usual procedure of assigning a value of the reflection coefficient f of about 0.9 in the so-called slip-flow regime is understandable, but this value of f is not consistent with the value of $f \cong 1$ found by Kuhlthau in the near-free molecule-flow regime. It is a task of future theoretical and experimental studies to clear up some of these anomalies. In this connection, it is possible that the spacing between the two cylinders could be reduced considerably, thereby permitting the same values of Re/M to be attained at higher pressures, where the experimental difficulties are less severe.

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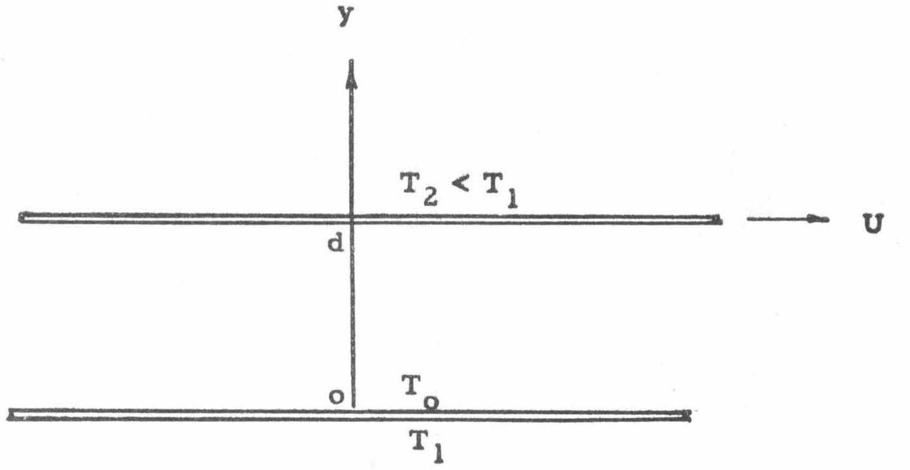

FIGURE 1

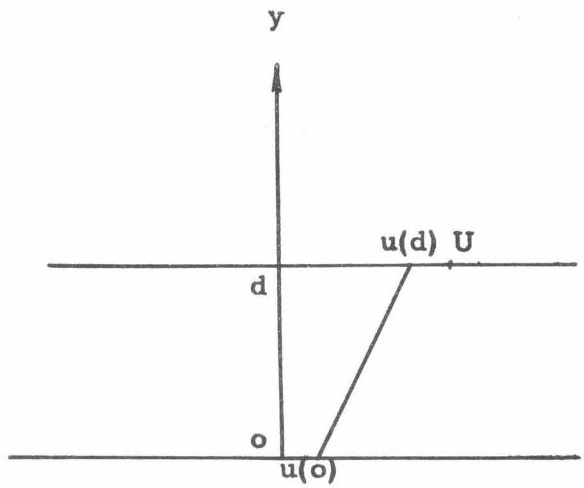

FIGURE 2

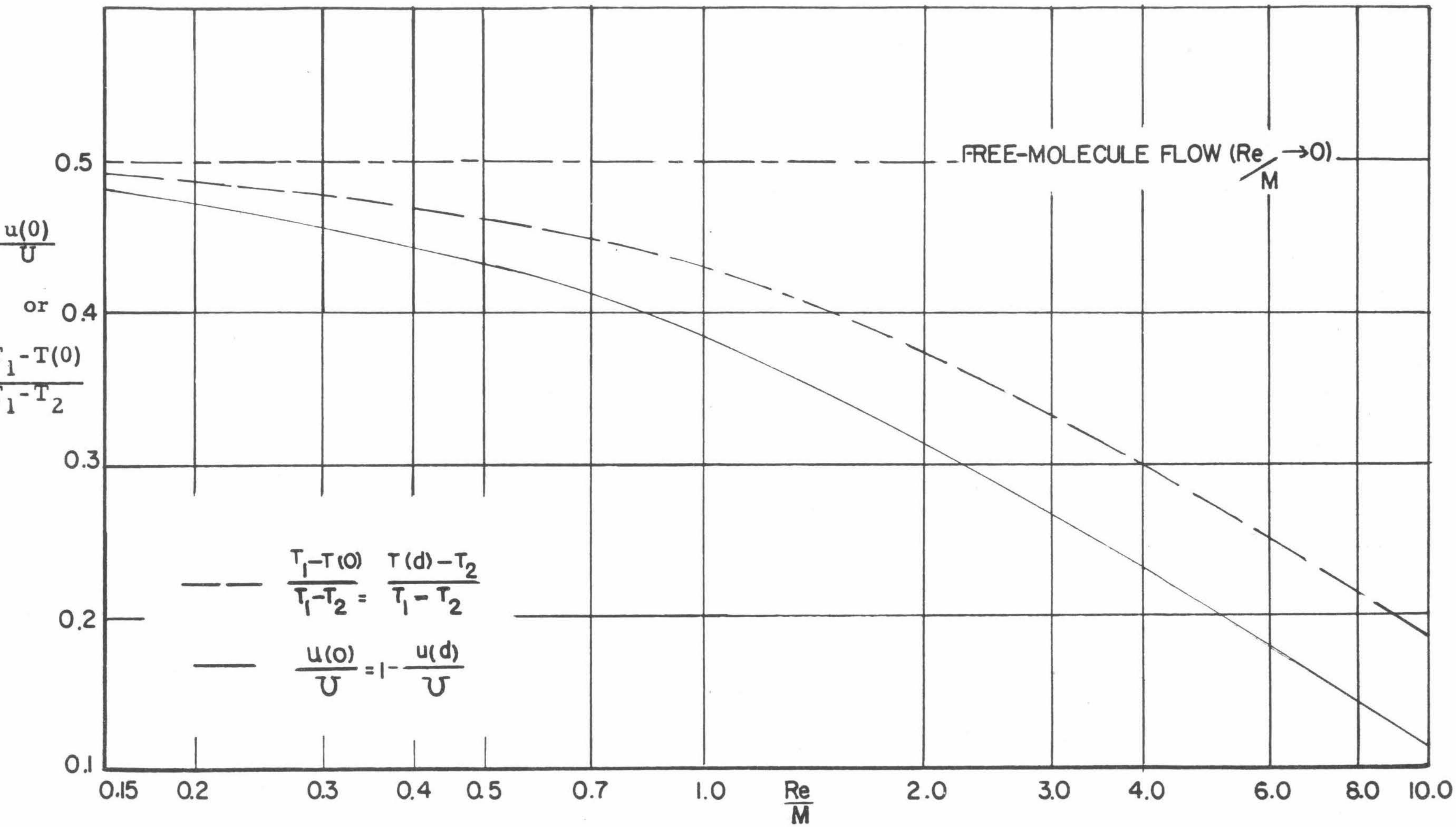

FIG. 3 SLIP VELOCITY AND TEMPERATURE JUMP AT PLATE SURFACE IN PLANE COUETTE FLOW (MONATOMIC GAS-DIFFUSE REFLECTION)

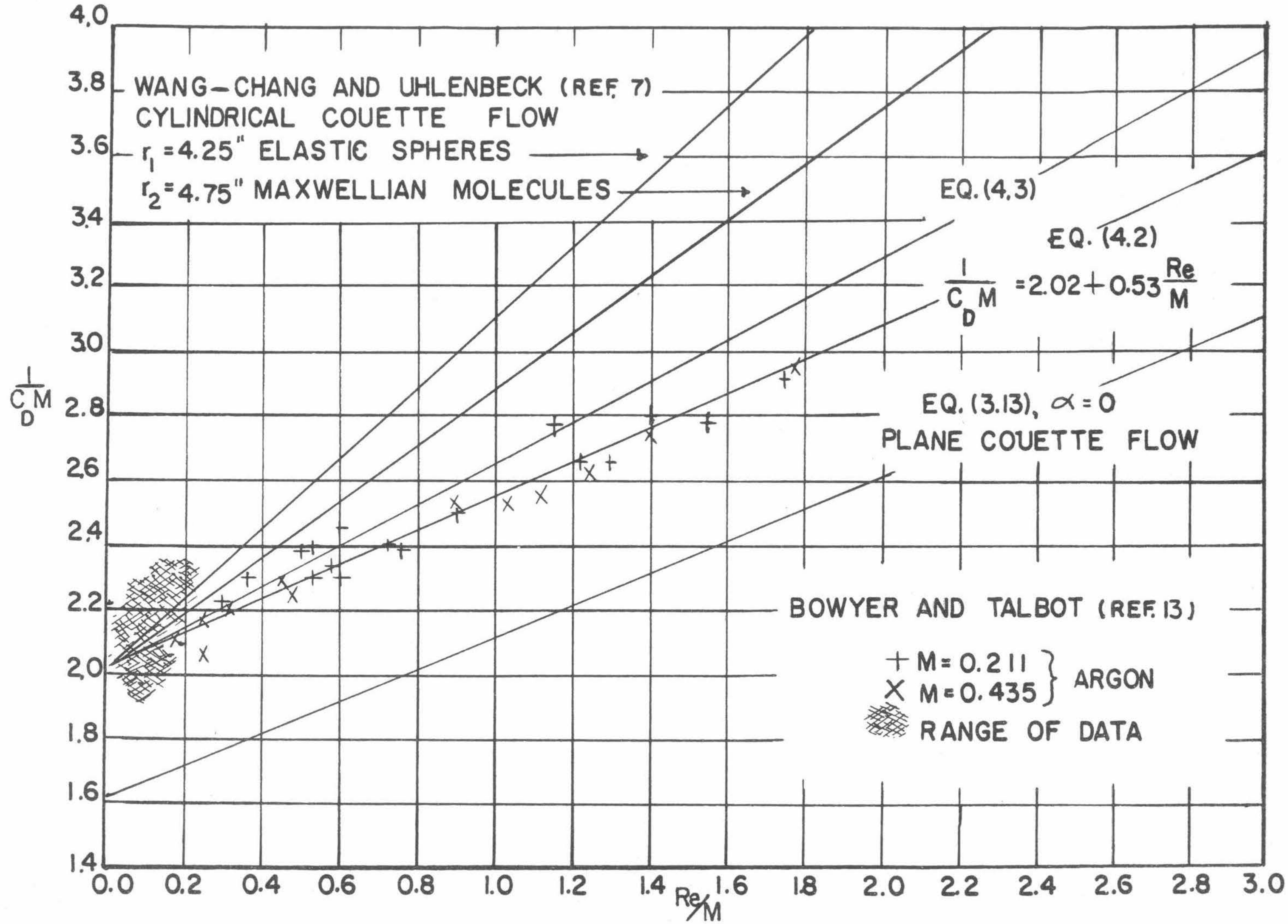

FIG. 4 DRAG COEFFICIENT ON PLATE SURFACE OR OUTER STATIONARY CYLINDER

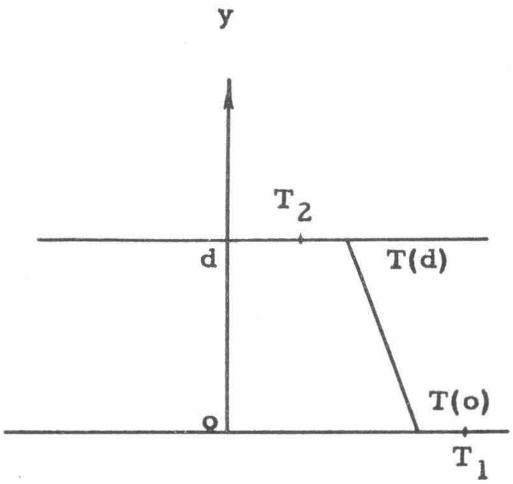

FIGURE 5

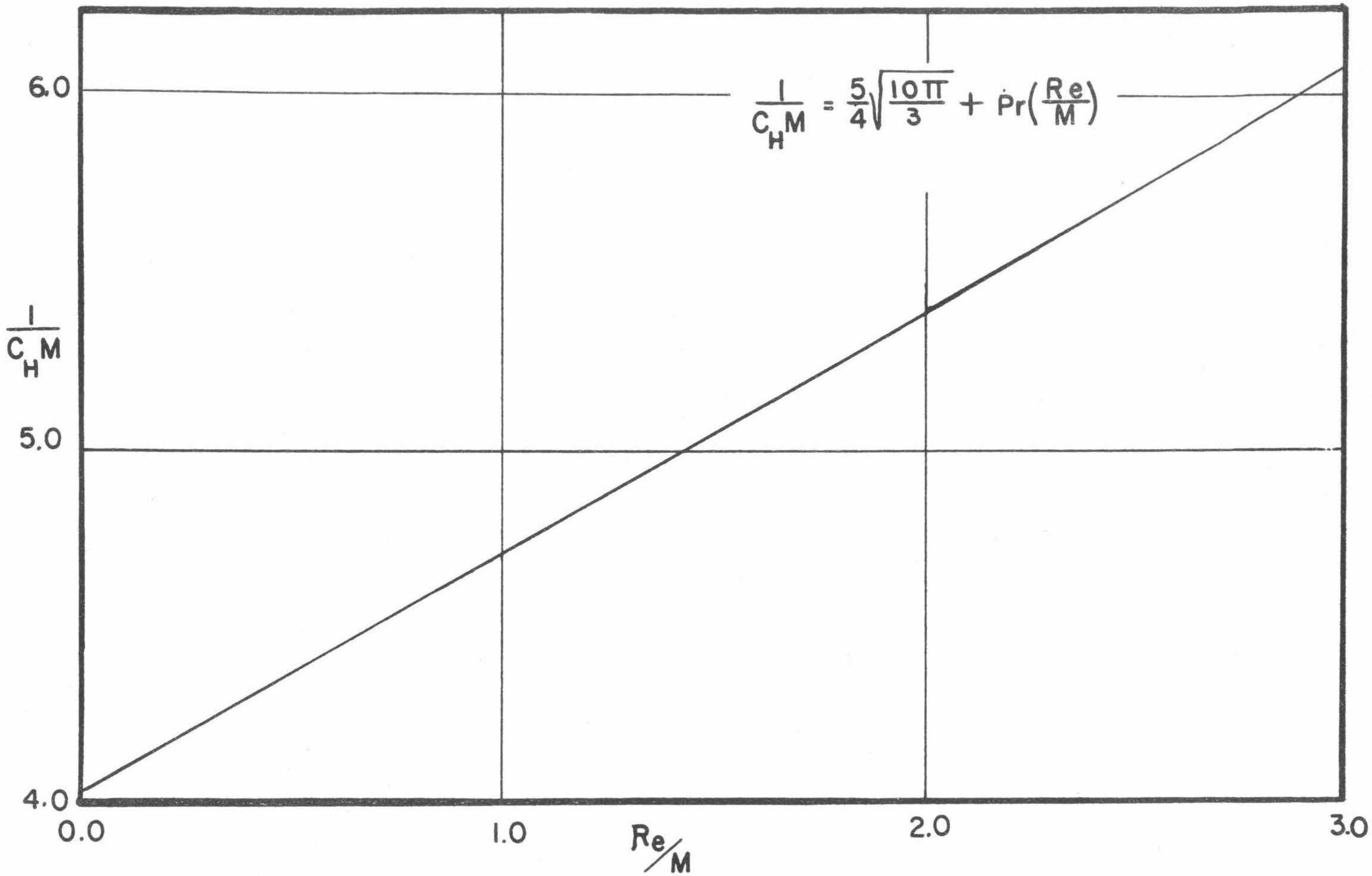

FIG. 6 STANTON NUMBER FOR PLANE COUETTE FLOW
(MONATOMIC GAS-DIFFUSE REFLECTION)

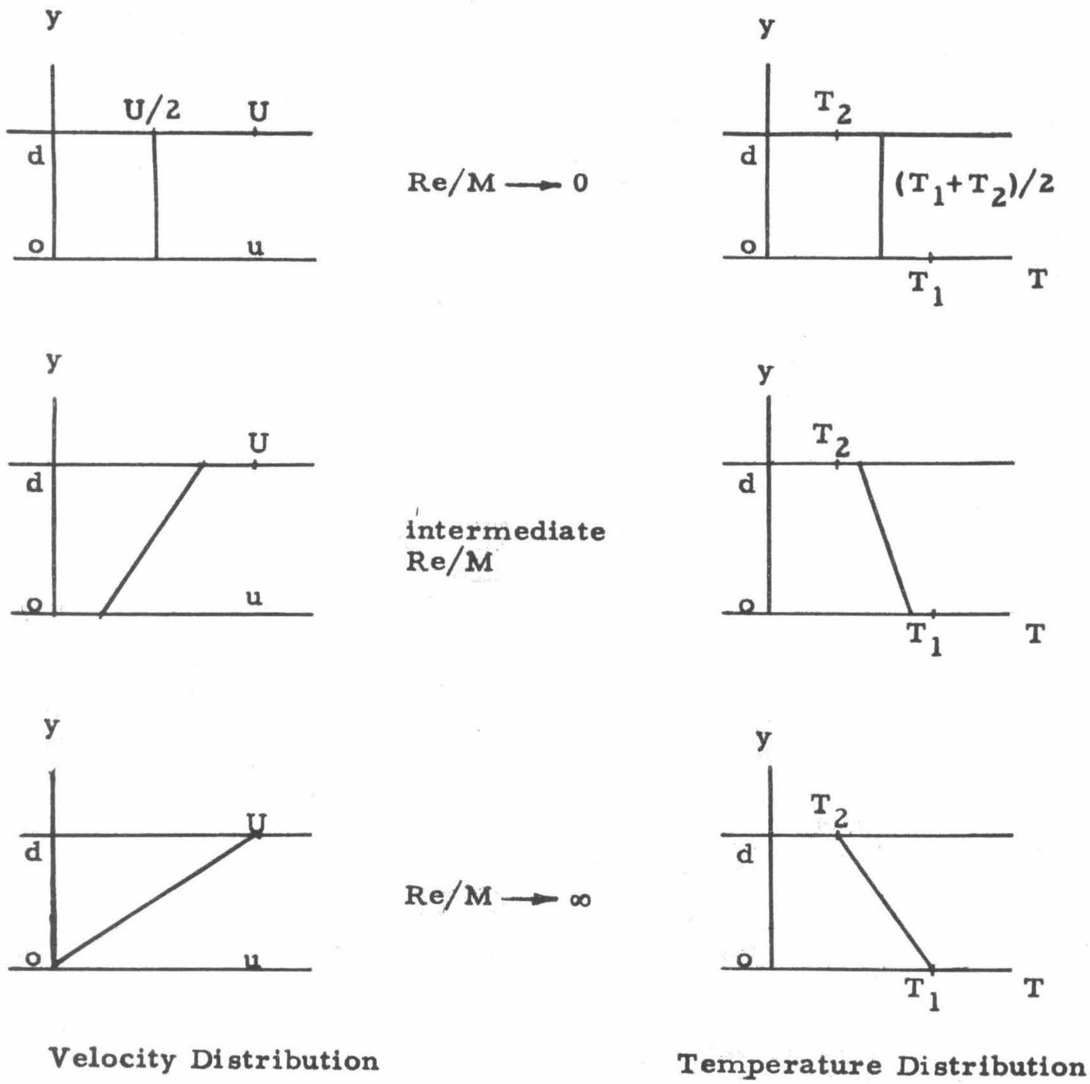
y
U/2
U
d
o
u
Re/M → 0
T
T_2
$(T_1+T_2)/2$
T_1
intermediate
Re/M
Re/M → ∞
Velocity Distribution
Temperature Distribution

FIGURE 7

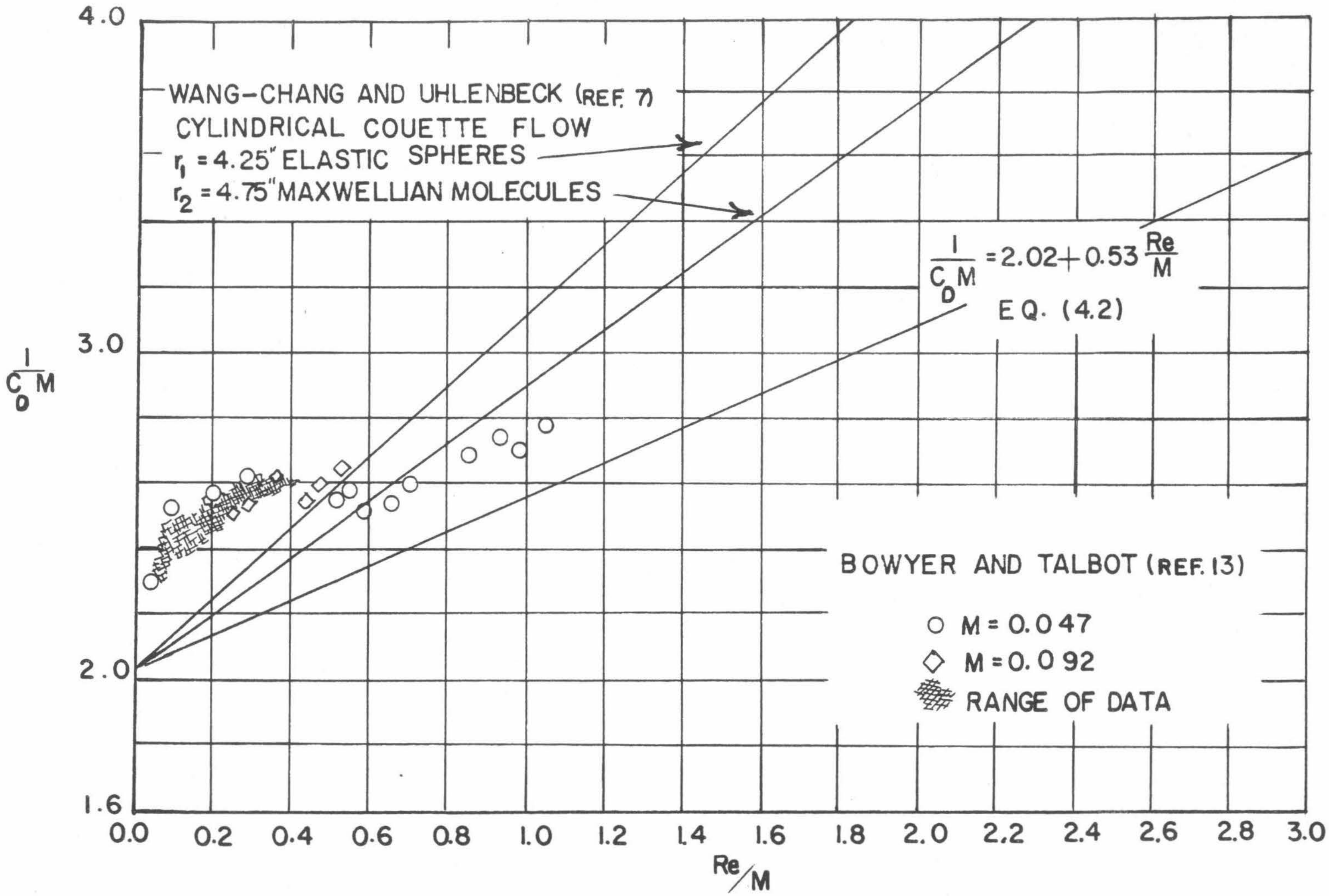

FIG. 8a DRAG ON STATIONARY OUTER CYLINDER IN COUETTE FLOW —— HELIUM

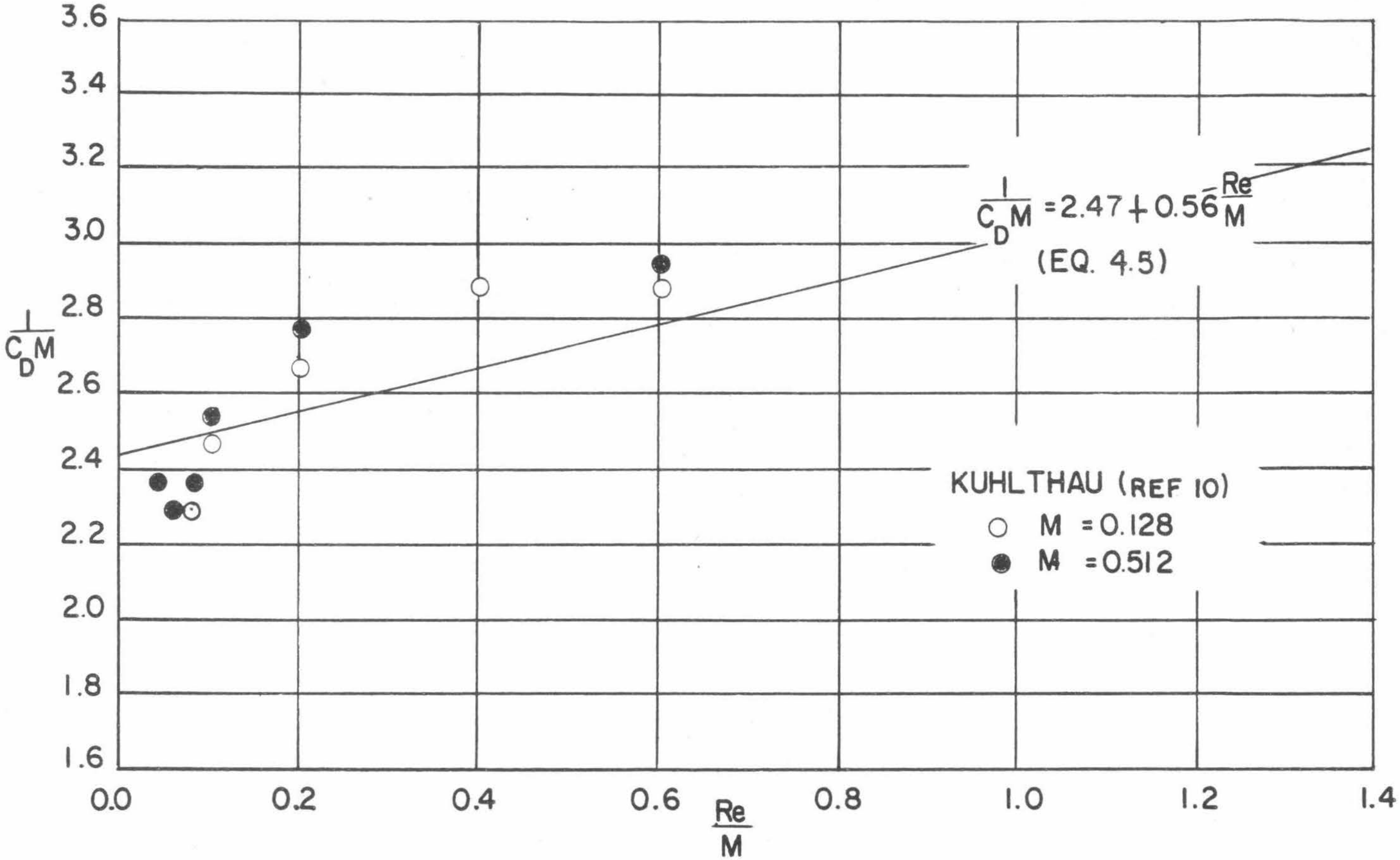

FIG. 8b DRAG ON STATIONARY OUTER CYLINDER IN COUETTE FLOW — HELIUM

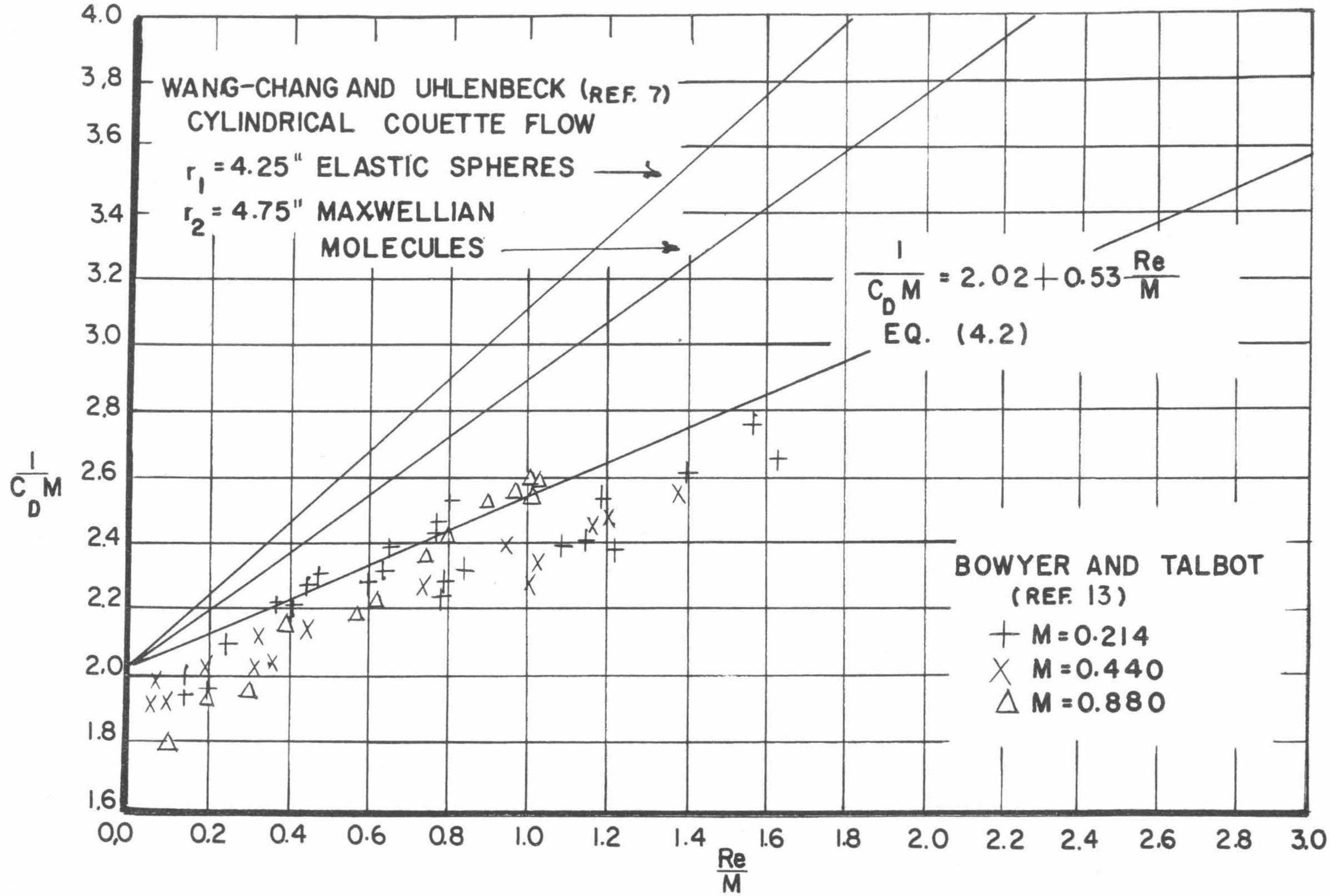

FIG. 9 DRAG ON STATIONARY OUTER CYLINDER IN COUETTE FLOW — KRYPTON

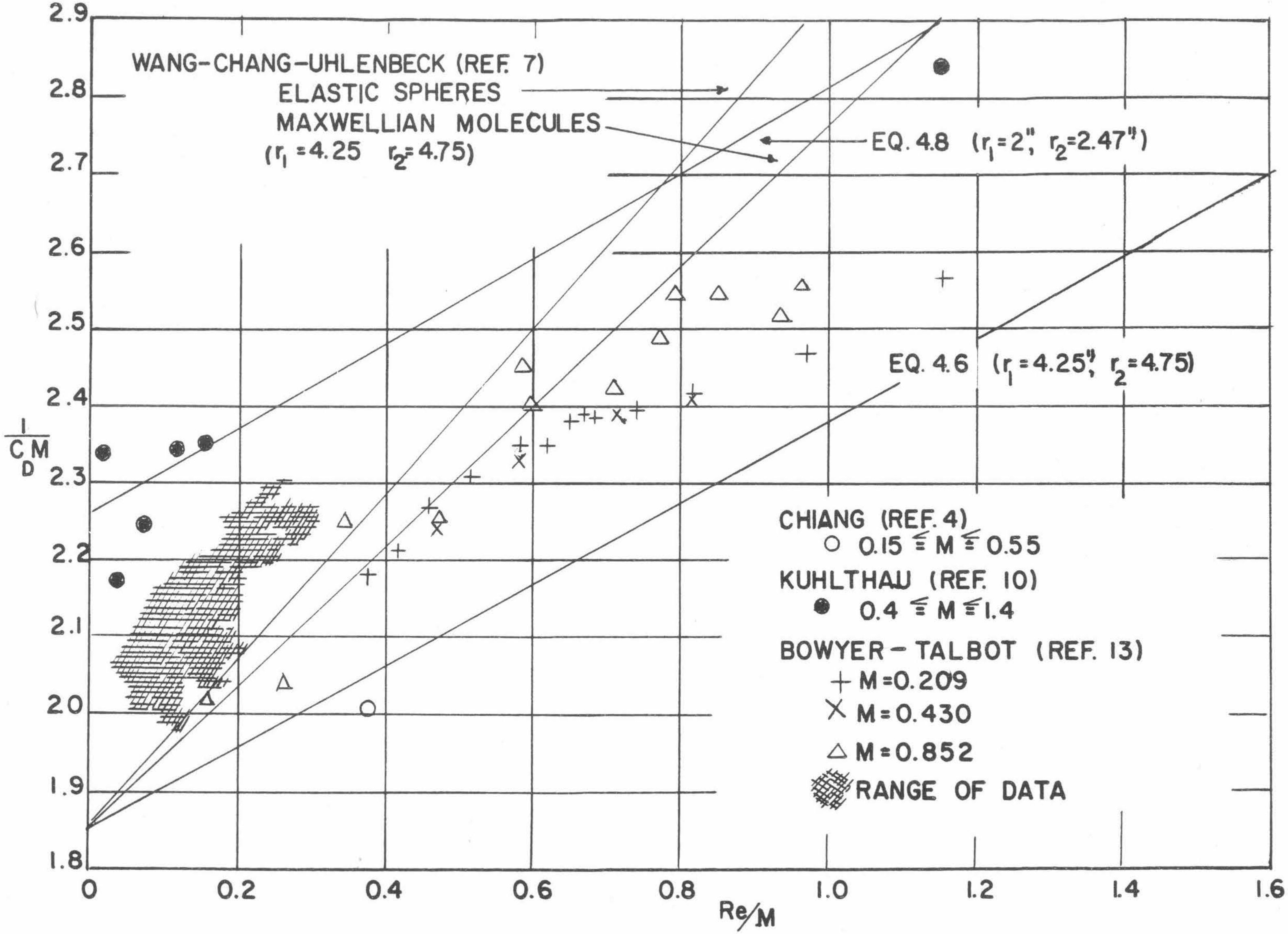

FIG. 10 DRAG ON STATIONARY OUTER CYLINDER IN COUETTE FLOW — AIR

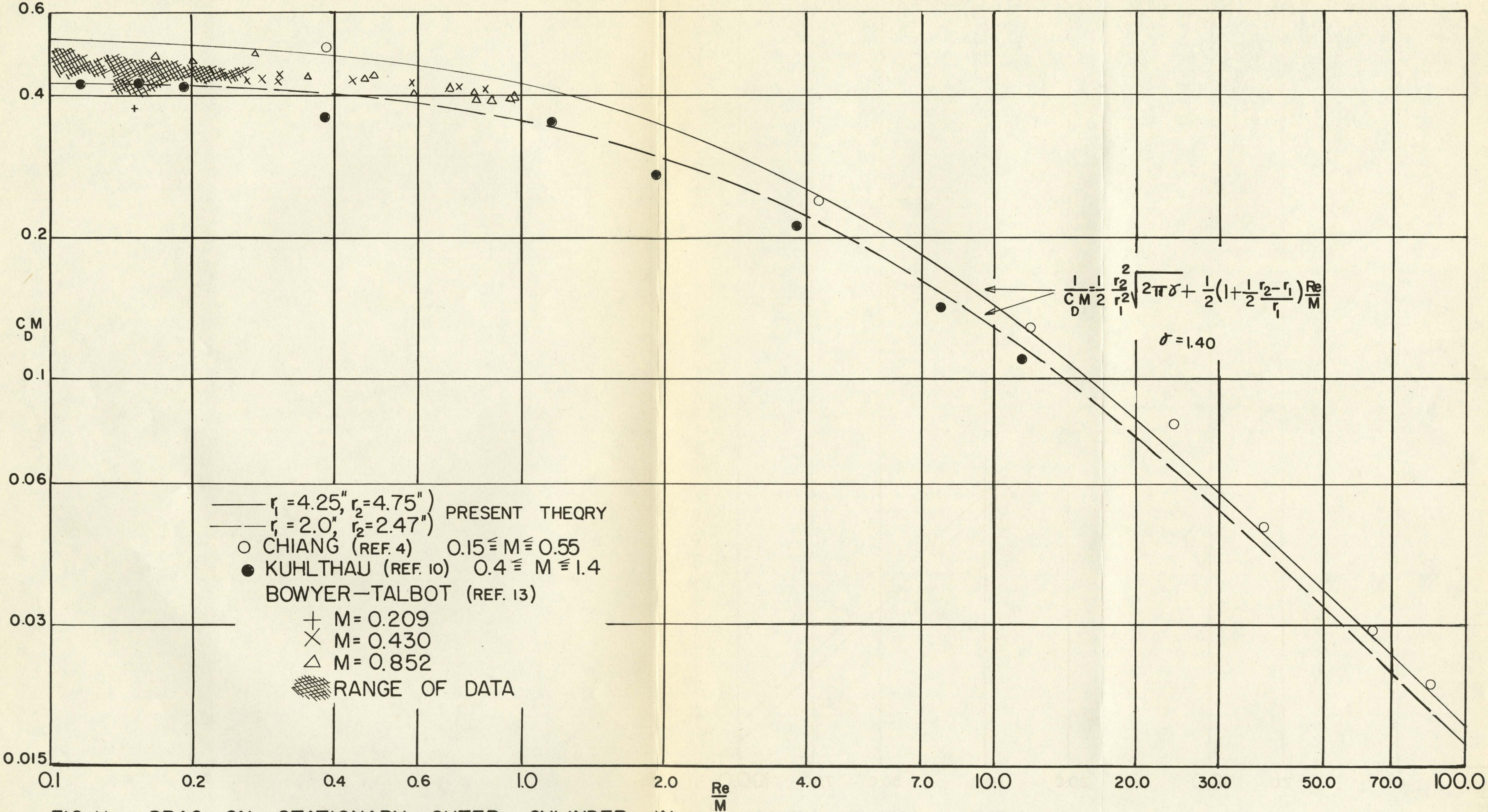

FIG. 11 DRAG ON STATIONARY OUTER CYLINDER IN COUETTE FLOW — AIR

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